COMPLEX ANALYTICAL METHODS IN THE THEORY OF LÉVY PROCESSES

ALEXEY KUZNETSOV

Complex analysis provides powerful tools for studying Levy processes. These tools can be used to solve many diverse problems, such as: finding the distribution of extrema, establishing the asymptotic behavior of the density of the overshoot, and computing the price of an Asian option in a Levy-driven model. In these lectures I will give a short introduction to Complex Analytical tools and then I will present a number of case-studies of how these tools can be applied to studying Levy processes.

These are some of the topics that we will discuss:
1) Analytic and meromorphic functions, analytic continuation, residues and Cauchy theorem, Laplace and Mellin transforms;
2) Finding the distribution of extrema for hyperexponential, meromorphic and stable Levy processes;
3) Computing the scale function for processes with completely monotone jumps;
4) Finding the distribution of the exponential functional.

If time permits, we will also talk about applications of these results in Mathematical Finance and Actuarial Mathematics.

References

[1] Jean Bertoin, *Levy processes,* Cambridge University Press, 1996.
[2] John B. Conway, *Functions of one complex variable I,* Second edition, Graduate Texts in Mathematics, Springer, 1978.
[3] Andreas E. Kyprianou, *Fluctuations of Levy processes with applications: introductory lectures,* Second edition, Springer, 2014.
[4] Dragoslav S. Mitrinovic and Jovan D. Keckic, *The Cauchy Method of Residues: Theory and Applications,* Volumes 1 and 2, Mathematics and its applications: East European series, Kluwer Academic Publishers, 1984 (Vol. 1) and 1993 (Vol. 2).