

Lévy 2016

8th International Conference on Lévy Processes 25.07.2016 - 29.07.2016 Angers, France

Program Booklet



Foreword

The organizing committee is pleased to welcome you to Angers for the international conference *Lévy 2016*.

The conference aims to bring together experts of different aspects of Lévy processes, encouraging them to exchange on their research and enabling them to share their knowledge with young researchers. Throughout the week, talks and posters will highlight recent advances in various fields of research which are currently studied. We hope that this conference will be a source of inspiration for your future research projects.

We also hope this conference will be an opportunity for you to visit Angers and its area. The cultural and historical heritage of the region Anjou is very rich and we encourage you to make the most of it during the week.

The organization of the conference is much indebted for the financial support of DéfiMaths, Université d'Angers, Laboratoire Angevin de Recherche en Mathématiques, Ministère de l'Enseignement Supérieur et de la Recherche, Centre National de la Recherche Scientifique, Région des Pays de la Loire, Centre de Mathématiques Henri Lebesgue, Angers Loire Métropole.

Finally, we are very grateful to all of you for attending and making the conference worthwhile by your contributions.

> Angers, July 2016 The organizing committee

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SCHEDULE

Monday, July 25th

8:15 – 8:30 Welcome

8:30 - 9:15 Víctor Rivero

Some fluctuation theory results for Markov additive processes and its applications to self-similar Markov processes

9:15 – 10:00 **Steffen Dereich** *MAPs, Kuznetsov measures and self-similarity*

10:00 – 10:45 **Batı Şengül** Entrance laws at the origin of self-similar Markov processes in \mathbb{R}^d

10:45 – 11:15 *Coffee break*

11:15 – 12:00 **Alexander Lindner** *On exponential functionals of Lévy processes*

12:00 – 12:45 **Mladen Savov** Bernstein-gamma functions and exponential functionals of Lévy processes

12:45 – 14:30 Lunch

14:30 – 15:15 **Philippe Marchal** *Complete subordinators with nested ranges*

15:15 – 16:00 **Wissem Jedidi** *New results related to complete monotonicity and Mellin transform, with applications to infinite divisibility*

16:00 – 16:30 Coffee break

16:30 – 17:15 **Nicolas Curien** *Lévy processes inside random planar maps*

17:15 – 18:00 **Bénédicte Haas** *A line-breaking construction of the stable trees*

18:00 – 18:45 **Gerónimo Uribe Bravo** *On splitting trees and Lévy processes*

Tuesday, July 26th

8:30 – 9:15 **Ron Doney** *The strong renewal theorem*

9:15 – 10:00 Vitali Wachtel First-passage times for random walks with non-identically distributed increments

10:00 – 10:45 Vladislav Vysotsky The absorption problem for multidimensional random walks

10:45 – 11:15 Coffee break

11:15 – 12:00 **Ewa Damek** *Recent developments in random affine and affine like recursions*

12:00 – 12:45 **Nikola Sandrić** *Markov chain approximation of pure jump processes*

12:45 - 14:30 Lunch

14:30 – 15:15 **Antoine Ayache** *Stationary increments harmonizable stable fields: upper estimates on path behavior*

15:15 – 16:00 Aleksandar Mijatović Invariance principle for random walks with anomalous recurrence properties

16:00 – 16:30 *Coffee break*

16:30 – 17:15 Kouji Yano Generalization of refracted Lévy processes and its application to exit problems

17:15 – 18:00 Kazutoshi Yamazaki On the refracted-reflected spectrally one-sided Lévy processes

18:30 – 19:30 Greetings from the local authorities (Jean Lurçat Museum)

Wednesday, July 27th

8:30 – 9:15 **Jacek Małecki** *On suprema of Lévy processes*

9:15 – 10:00 Mateusz Kwaśnicki Non-symmetric Lévy processes: suprema and eigenfunctions

10:00 – 10:45 **Tomasz Grzywny** *Transition densities of isotropic unimodal Lévy processes (asymptotics and estimates)*

10:45 – 11:15 *Coffee break*

11:15 – 12:00 **Panki Kim** Dirichlet heat kernel estimates for symmetric Lévy processes

12:00 – 12:45 **Jean-Christophe Breton** *Asymptotics in random balls models*

12:45 - 14:30 Lunch

14:30 - Conference excursion

Thursday, July 28th

8:30 - 9:15 **Jean Jacod**

Estimating high activity jumps for discretely observed processes with noisy and irregular observations

9:15 – 10:00 Peter Tankov

Optimal importance sampling for Lévy processes

10:00 – 10:45 José-Luis Pérez-Garmendia

Spectrally negative Lévy processes with Parisian reflection below and classical reflection above

10:45 – 11:15 *Coffee break*

11:15 – 12:00 Boris Buchmann

Distributional representations and dominance of a Lévy process over its maximal jump processes

12:00 – 12:45 Juan Carlos Pardo Millán

Branching processes in a Lévy random environment

 $12{:}45-14{:}30\ Lunch$

14:30 – 15:15 **Robert Stelzer**

Lévy-driven CARMA processes: Non-equidistant observations and local stationarity

15:15 – 16:00 **Andreas Basse-O'Connor** *Limit theorems for a class of stationary increments Lévy driven moving averages*

16:00 - 16:30 Coffee break

16:30 – 17:15 **Anita Behme** *Invariant distributions of Itô-Lévy processes*

17:15 – 18:00 **Bert Zwart** Sample-path large deviations for heavy-tailed random walks and Lévy processes

19:30 – Conference dinner

Friday, July 29th

8:30 – 9:15 **Yanxia Ren** *Williams decomposition for superprocesses*

9:15 – 10:00 **Leif Döring** *On perpetual integrals*

10:00 – 10:45 **Martijn Pistorius** *On weak approximation of BSDEs driven by Lévy processes*

10:45 – 11:15 *Coffee break*

11:15 – 12:00 Markus Riedle Stochastic integration with respect to cylindrical Lévy processes

12:00 – 12:45 **Alex Watson** *Fragmentation with growth*

12:45 - 14:30 Lunch

TALKS

Antoine Ayache (Université de Lille 1)

Studying sample path behavior of stochastic fields/processes is a classical research topic in probability theory and related areas such as fractal geometry. To this end, many methods have been developed for a long time in Gaussian frames. They often rely on some underlying "nice" Hilbertian structure, and can also require finiteness of moments of high order. Therefore, they can hardly be transposed to frames of heavy-tailed stable probability distributions. However, in the case of some linear non-anticipative moving average stable fields/processes, such as the linear fractional stable sheet and the linear multifractional stable motion, rather new wavelet strategies have already proved to be successful in order to obtain sharp moduli of continuity and other results on sample path behavior. The main goal of our talk is to show that, despite the difficulties inherent in the frequency domain, such kind of a wavelet methodology can be generalized and improved, so that it also becomes fruitful in a general harmonizable stable setting with stationary increments. Let us point out that there are large differences between this harmonizable setting and the moving average stable one.

The talk is based on joint work with Geoffrey Boutard (Université de Lille 1)

Limit theorems for a class of stationary increments Lévy driven moving averages

Andreas Basse-O'Connor (Aarhus University)

In this talk we present some new limit theorems for power variation of *k*th order increments of stationary increments Lévy driven moving averages. In the infill asymptotic setting, where the sampling frequency converges to zero while the time span remains fixed, the asymptotic theory gives very surprising results, which (partially) have no counterpart in the theory of discrete moving averages.

More specifically, we show that the first order limit theory and the mode of convergence strongly depend on the interplay between the given order of the increments $k \ge 1$, the considered power p > 0, the Blumenthal–Getoor index $\beta \in (0,2)$ of the driving pure jump Lévy process *L* and the behaviour of the kernel function *g* at 0 determined by the power α .

First order asymptotic theory essentially comprises three cases: stable convergence towards a certain infinitely divisible distribution, an ergodic type limit theorem and convergence in probability towards an integrated random process. We also prove a second order limit theorem connected to the ergodic type result. When the driving Lévy process *L* is a symmetric β -stable process we obtain two different limits: a central limit theorem and convergence in distribution towards a $(k - \alpha)\beta$ -stable totally right skewed random variable.

This talk is based on joint work with Raphaël Lachièze-Rey (Paris Descartes University) and Mark Podolskij (Aarhus University).

Invariant distributions of Itô-Lévy processes

Anita Behme

(Otto-von-Guericke-Universität Magdeburg)

We present a new integral criterion for the existence of an invariant measure of an Itô-Lévy process. This criterion is based on the probabilistic symbol of the Itô-Lévy process. In contrast to the standard integral criterion for invariant measures of Markov processes based on the generator, no test functions and hence no information on the domain of the generator is needed.

Further, we discuss a possible extension of this criterion to avoid certain moment conditions. This extension is based on a one-sided symbol of one-sided bounded Itô-Lévy processes, that we will introduce.

This talk is based on joint work with Alexander Schnurr (Siegen University).

References

- [1] A. Behme and A. Schnurr, *A criterion for invariant measures of Itô processes based on the symbol.* Bernoulli 21 (2015), no. 3, 1697-1718
- [2] A. Behme and A. Schnurr, *A one-sided symbol for Itô-Lévy processes*. Submitted

Asymptotics in random balls models

Jean-Christophe Breton (Université de Rennes 1)

Random balls models are collections of Euclidean balls whose centers and radii are generated by a Poisson point process. Such collections model various contexts ranging from imaging to communication network. When the distributions driving the centers and the radii are heavy-tailed, interesting interference phenomena occurs when the model is properly zoomed-out.

The talk aims to illustrate such phenomena and to give an overview of the asymptotic behavior of functionals of interest. The limits obtained include in particular stable fields, (fractional) Gaussian fields and Poissonian bridges. Related questions will also be discussed.

Distributional representations and dominance of a Lévy process over its maximal jump processes

Boris Buchmann

(Australian National University)

Distributional identities for a Lévy process X_t , its quadratic variation process V_t , and its maximal jump processes, are derived, and used to make "small time" (as $t \downarrow 0$) asymptotic comparisons between them. The representations are constructed using properties of the underlying Poisson point process of the jumps of X. Apart from providing insight into the connections between X, V, and their maximal jump processes, they enable investigation of a great variety of limiting behaviours. As an application we study "self-normalized" versions of X_t , that is, X_t after division by $\sup_{0 < s \le t} \Delta X_s$, or by $\sup_{0 < s \le t} |\Delta X_s|$. Thus we obtain necessary and sufficient conditions for $X_t / \sup_{0 < s \le t} \Delta X_s$ and $X_t / \sup_{0 < s \le t} |\Delta X_s|$ to converge in probability to 1, or to ∞ , as $t \downarrow 0$, so that X is either comparable to, or dominates, its largest jump. The former situation tends to occur when the singularity at 0 of the Lévy measure of X is fairly mild (its tail is slowly varying at 0), while the latter situation is related to the relative stability or attraction to normality of X at 0 (a steeper singularity at 0). An important component in the analyses is the way the largest positive and negative

jumps interact with each other. Analogous "large time" (as $t \to \infty$) versions of the results can also be obtained.

Joint work with Yuguang Fan (University of Melbourne) and Ross A. Maller (Australian National University).

Lévy processes inside random planar maps

Nicolas Curien (Université Paris-Sud Orsay)

We will show that the study of the geometric structure of large random planar maps such as triangulations involves many stable laws and Lévy processes. A particular role is played by the spectrally negative stable Lévy process of index 3/2. We will show how these processes pop-up inside random planar maps and how to use them to study for example the volume growth inside random planar maps.

Recent developments in random affine and affine like recursions

Ewa Damek (University of Wrocław)

We consider the following affine recursion in \mathbb{R}

$$X_n = A_n X_{n-1} + B_n, \quad n \ge 1 \tag{1}$$

where (A_n, B_n) is a sequence of i.i.d. (independent identically distributed) random variables with values in $\mathbb{R} \times \mathbb{R}$ and $X_0 \in \mathbb{R}$ is the initial distribution. Under mild contractivity hypotheses the sequence X_n converges in law to a random variable R, which is the unique solution of the stochastic difference equation

 $R =_d AR + B$, where *R* is independent of (A, B)

and equality is meant in law. The main issues concerning (1) are characterization of the tail of R, regularity of the law of R, behavior of iterations X_n .

Under appropriate assumptions, the sequence $X_1 + \cdots + X_n$ normalized properly converges in law to a stable variable. When the law of (A_1, B_1) comes from

an appropriate diffusion semigroup the chain X_n is embedded in a generalized Ornstein-Uhlenbeck process of Behme-Lindner and the links will be discussed.

With so called Kesten assumptions *R* has a heavy tail behavior, which means that there is $\alpha > 0$ such that $\lim_{t\to\infty} \mathbb{P}(R > t) t^{\alpha} = C_+$ and $\lim_{t\to\infty} \mathbb{P}(R < -t) t^{\alpha} = C_-$. While $C_+ + C_-$ is always positive provided the iterations are not constant, conditions for positivity of C_+ , C_- are not that trivial.

I am going to address the latter question both in the case of the above recursion and Lipschitz recursions modeled on it and give a very simple proof of positivity of C_+ . The talk is based on the joint work with Dariusz Buraczewski but the previous results of Guivarc'h and Le Page as well as Mirek will be referred to.

MAPs, Kuznetsov measures and self-similarity

Steffen Dereich (University of Münster)

Since the seminal work of Lamperti there is a lot of interest in the understanding of the general structure of self-similar Markov processes. Lamperti gave a representation of positive self-similar Markov processes with initial condition strictly larger than 0 which subsequently was extended to zero initial condition.

For real self-similar Markov processes (rssMps) there is a generalization of Lamperti's representation giving a one-to-one correspondence between Markov additive processes and rssMps with initial condition different from the origin.

We develop fluctuation theory for Markov additive processes and use Kuznetsov measures to construct the law of transient real self-similar Markov processes issued from the origin. The construction gives a pathwise representation through two-sided Markov additive processes extending the Lamperti-Kiu representation to the origin.

The strong renewal theorem

Ron Doney (University of Manchester)

For over 50 years the following question has been open: when does the local version of the renewal theorem hold in the case of renewal processes, or more generally random walks, which are in the domain of attraction of a stable law of index $0 < \alpha < 1$, where in the case of a random walk it is assumed additionally that the positivity parameter is positive?

The result holds without any further assumptions in case $\alpha > 1/2$, but it turns out that in all other cases the NASC is quite complicated, especially for random walks. The proof relies on some simple, but new large deviation estimates.

This is joint work with Francesco Caravenna.

On perpetual integrals

Leif Döring (Universität Mannheim)

We discuss a 0-1 law on perpetual integrals of Lévy processes and applications to Jump SDEs.

Transition densities of isotropic unimodal Lévy processes (asymptotics and estimates)

Tomasz Grzywny (Wrocław University of Science and Technology)

One of the well-known results of the asymptotic behaviour of the transition density of pure-jump Lévy processes originates from the twenties of the last century (Pólya (1923), Blumenthal, Getoor (1961)) and it concerns the transition density $p_t^{(\alpha)}(x)$ of the symmetric α -stable process X_{α} in \mathbb{R}^d ,

$$\lim_{x \to \infty} |x|^{d+\alpha} p_1^{(\alpha)}(x) = \alpha 2^{\alpha-1} \pi^{-d/2-1} \sin\left(\frac{\alpha \pi}{2}\right) \Gamma\left(\frac{\alpha+d}{2}\right) \Gamma\left(\frac{\alpha}{2}\right).$$
(2)

Applying the remarkable scaling property of the stable process in \mathbb{R}^d , that is $p_t^{(\alpha)}(x) = t^{-d/\alpha} p_1^{(\alpha)}(1, t^{-1/\alpha}x)$, we obtain the similar asymptotic expression for $p_t^{(\alpha)}(x)$ as $t^{-1/\alpha}x$ goes to infinity.

In this talk we investigate such behaviour for transition densities of isotropic unimodal Lévy processes. Let $\mathbf{X} = (X_t : t \ge 0)$ be an isotropic unimodal Lévy process on \mathbb{R}^d with the characteristic exponent ψ . In the recent paper [1], the estimates of transition densities p(t, x) were studied under the assumption that ψ has lower and upper Matuszewska indices stritly between 0 and 2. More detailed information can be obtained whenever stronger asumption about the behaviour of ψ is imposed. Namely, in [2], it is proved that if ψ varies regulary at inifinity with index $\alpha \in (0, 2)$ then

$$\lim_{(x,t\psi(|x|^{-1}))\to 0} \frac{p(t,x)}{t|x|^{-d}\psi(|x|^{-1})} = \mathscr{A}_{d,\alpha}.$$
(3)

Moreover, the asymptotic (3) implies that ψ varies regularly at initiality with index $\alpha \in (0,2)$.

The natural question arises about the asymptotic behaviour and estimates of the semigroup for the endpoints $\alpha \in \{0, 2\}$. In this talk we shed some light on the situation $\alpha = 0$. We provide a solution for a large class of slowly varying symbols including geometric stable and iterated geometric stable cases. Namely, we study processes with symbols belonging to de Haan class Π_{ℓ}^{∞} associated to a function ℓ slowly varying at inifinity which is equivalent to regular variation with index -d at the origin of the density of Lévy's measure of the process. We derive several asymptotic results describing the behaviour of transition densities p(t, x). Moreover, we present several estimates for p(t, x) under various assumptions for isotropic unimodal processes, which improve the existing results even in the case of subordinate Brownian motion (see [3]).

Joint works with Wojciech Cygan, Michał Ryznar and Bartosz Trojan.

References

[1] Krzysztof Bogdan, Tomasz Grzywny and Michał Ryznar, *Density and tails of unimodal convolution semigroups*. J. Funct. Anal. 266 (2014), no. 6, 3543–3571.

- [2] Wojciech Cygan, Tomasz Grzywny and Bartosz Trojan, *Asymptotic behavior of densities of unimodal convolution semigroups*. To appear in Trans. Amer. Math. Soc.
- [3] Tomasz Grzywny, Michał Ryznar and Bartosz Trojan, *Asymptotic behaviour and estimates of slowly varying convolution semigroups*. Preprint 2016.

A line-breaking construction of the stable trees

Bénédicte Haas (Université Paris 13)

We give a new, simple construction of the α -stable tree for $\alpha \in (1,2]$. We obtain it as the closure of an increasing sequence of \mathbb{R} -trees inductively built by gluing together line-segments one by one. The lengths of these line-segments are related to the increments of an increasing \mathbb{R}_+ -valued Markov chain. For $\alpha = 2$, we recover Aldous' line-breaking construction of the Brownian continuum random tree based on an inhomogeneous Poisson process. Based on a joint work with Christina Goldschmidt (Oxford).

Estimating high activity jumps for discretely observed processes with noisy and irregular observations

Jean Jacod (Université Pierre-et-Marie-Curie)

We will consider a 1-dimensional process which is observed at irregularly spaced times, within a fixed time interval, and with an observation noise (a typical setting for price or log-price processes in financial mathematics). Assuming that the process has "high-activity" jumps, meaning its Blumenthal-Getoor index is positive, and perhaps close to 2, our aim is to estimate this degree of activity and the corresponding "intensity process". For this we use a novel method, based on de-noising first, and then averaging some local empirical characteristic functions of the increments of the process.

This is a joint work with Viktor Todorov, from Northwestern University.

New results related to complete monotonicity and Mellin transform, with applications to infinite divisibility

Wissem Jedidi (King Saud University)

In a first part, we present several new characterizations of completely monotone functions and Bernstein functions via two approaches: the first one is driven algebraically via elementary preserving mappings and the second one is developed in terms of the behavior of their restriction on \mathbb{N} . We give a complete answer to the following question: *Can we affirm that a function f is completely monotone (respectively a Bernstein function) if we know that the sequence* $(f(k))_k$ is completely monotone (respectively alternating)? This approach constitutes a kind of converse to Hausdorff's moment characterization theorem in the context of completely monotone sequences. In a second part, we present new properties for the Mellin transform of nonnegative random variables related to injectivity, simplification and characterization of the convergence in distribution. These tools allow us to refine a limit theorem obtained by Harkness and Shantaram (1969) on families obtained by biasing nonnegative random variables.

Dirichlet heat kernel estimates for symmetric Lévy processes

Panki Kim (Seoul National University)

In this talk, we discuss the behavior of transition density (Dirichlet heat kernel) for symmetric Lévy processes in open subset. Recently there has been intense interest on studying the behavior of transition density of such processes. In this talk, after a survey of recent devolvement on this direction, I will discuss explicit sharp two-sided estimates for the Dirichlet heat kernels of a large class of symmetric Lévy processes.

Non-symmetric Lévy processes: suprema and eigenfunctions

Mateusz Kwaśnicki (Wrocław University of Technology)

I will present a result that links two objects: the distribution of the supremum of a Lévy process, and eigenfunctions for its generator in positive and negative half-lines. The process is assumed to have completely monotone jumps, and certain balance between positive and negative jumps is required. There is, however, no need to assume symmetry. In particular, our result covers nonsymmetric stable Lévy processes.

The symmetric case was studied jointly with Jacek Małecki and Michał Ryznar. In this case an explicit (though rather complicated) integral expression for the distribution of the supremum is found. A version for (also non-symmetric) stable processes is a recent joint result with Alexey Kuznetsov. The result extends further to processes with completely monotone jumps, under the conditions mentioned above (this part is still work-in-progress).

Curiously, when there are more positive than negative jumps, one of the integrals in our expression for the supremum requires Lindelöf-type summation: due to exponential growth of the eigenfunctions, the integral is not even conditionally convergent.

The above results require either heavy use of special functions, or rather involved analysis of certain complex functions. During my talk I will avoid these technicalities as far as possible, and I will focus on the core result and possible applications.

On exponential functionals of Lévy processes

Alexander Lindner (Ulm University)

Let ξ be a fixed one-dimensional Lévy process and consider the mapping

$$\begin{split} \Phi_{\xi} : & D_{\xi} \to \quad \text{set of probability distributions on } \mathbb{R}, \\ & \mathscr{L}(\eta_1) \mapsto \quad \mathscr{L}\left(\int_0^\infty e^{-\xi_{s-}} d\eta_s\right), \end{split}$$

where $\mathscr{L}(\cdot)$ denotes the law of a random variable and $D_{\xi} := \{\mathscr{L}(\eta_1) : (\eta_t)_{t \ge 0} \}$ 1-dim. Lévy process, independent of η , s.t. $\int_0^\infty e^{-\xi_{s-1}} d\eta_s$ converges a.s. $\}$.

The random variable $\int_0^\infty e^{-\xi_{s-}} d\eta_s$, whenever it exists, is called the *exponential functional* of the bivariate Lévy process (ξ, η) . Its distribution appears in various applications, e.g. as the stationary solution of generalized Ornstein–Uhlenbeck processes. In this talk, we shall be interested in properties of the mapping Φ_{ξ} , which associates to every η the distribution of the corresponding exponential functional when ξ is fixed. While the case $(\xi_t = t)_{t\geq 0}$ has been well studied and gives rise to all self-decomposable distributions, less is known for general ξ . In this talk, we shall consider the case of general ξ , and show that the range of Φ_{ξ} is closed under weak convergence, that Φ_{ξ} is injective for many cases of ξ , we consider continuity properties of Φ_{ξ} and its inverse, and study for which Lévy processes $\Phi_{\xi}(\mathcal{L}(\eta_1))$ is a standard normal distribution or a positive α -stable distribution.

The talk is based on joint works with Anita Behme and Makoto Maejima [1,2].

References

- A. Behme and A. Lindner, *On exponential functionals of Lévy processes*. J. Theoret. Probab. 28 (2015), 681–720.
- [2] A. Behme, A. Lindner and M. Maejima (2016+), *On the range of exponential functionals of Lévy processes.* Séminaire de Probabilités, to appear.

On suprema of Lévy processes

Jacek Małecki (Wrocław University of Technology)

Let us consider a real valued Lévy process X and its past supremum before any deterministic time t, i.e. $\overline{X}_t = \inf\{X_s : 0 \le s \le t\}$. The aim of the talk is to present some results describing the distribution of \overline{X}_t by providing estimates for the cumulative distributional function $\mathbb{P}(\overline{X}_t < x)$, formula for the Laplace transform of \overline{X}_t (for fixed t) in the symmetric case and the formula for $\mathbb{P}(\overline{X}_t < x)$ for subordinate Brownian motions. Moreover, we examine the asymptotic behaviour together with some regularity properties of the density $f_t(x) = \mathbb{P}(\overline{X}_t \in dx)/dx$ in cases where transition probabilities of *X* are absolutely continuous and have bounded densities.

These are joint works with L. Chaumont, M. Kwaśnicki and M. Ryznar.

References

- [1] L. Chaumont, J. Małecki, *On the asymptotic behavior of the density of the supremum of Levy processes*. Ann. Inst. H. Poincaré Probab. Statist., to appear (2016).
- [2] M. Kwaśnicki, J. Małecki and M. Ryznar, *First passage times for subordinate Brownian motions*. Stochastic Process. Appl. 123 (2013), no. 5, 1820–1850.
- [3] M. Kwaśnicki, J. Małecki and M. Ryznar, *Suprema of Lévy processes*. Ann. Probab. 41 (2013), no. 3B, 2047–2065.

Complete subordinators with nested ranges

Philippe Marchal (Université Paris 13)

We give a coupled construction of regenerative sets \mathscr{R}_f , indexed by all complete Bernstein functions f. For every f, the set \mathscr{R}_f is the range of a subordinator with Lévy-Khintchine exponent f and if $f \leq g$, then $\mathscr{R}_f \subset \mathscr{R}_g$. Similar constructions for more general special subordinators are also discussed.

Invariance principle for random walks with anomalous recurrence properties

Aleksandar Mijatović (King's College London)

We consider a class of spatially non-homogeneous random walks in multidimensional Euclidean space with zero drift, which in any dimension (two or higher) can be recurrent or transient depending on the details of the walk. These walks satisfy an invariance principle, and have as their scaling limits a class of martingale diffusions, with law determined uniquely by an SDE with discontinuous coefficients at the origin. Furthermore, pathwise uniqueness of this SDE may fail. The radial coordinate of the diffusion is a Bessel process of dimension greater than 1 (this component of the invariance principle is related to a theorem of Lamperti). Unique characterization in law of the diffusion, which must start at the origin, is natural via excursions built around the Bessel process; each excursion has a generalized skew-product-type structure, in which the angular component spins at infinite speed at the start and finish of each excursion. Defining appropriately the Remannian metric g on the sphere S^{d-1} allows us to give an explicit construction of the angular component (and hence of the entire skew-product decomposition) as a time-changed Browninan motion with drift on the Riemannian manifold (S^{d-1}, g) . For example, the density of the stationary law of the angular component with respect to the volume element of g can be characterised by a linear PDE involving the Laplace-Beltrami operator and the divergence under the metric g. This is joint work with Nicholas Georgiou and Andrew Wade.

Branching processes in a Lévy random environment

Juan Carlos Pardo Millán (CIMAT Guanajuato)

In this talk, we introduce branching processes in a Lévy random environment. In order to define this class of processes, we study a particular class of non-negative stochastic differential equations driven by Brownian motions and Poisson random measures which are mutually independent. The existence and uniqueness of strong solutions are established under some general conditions that allows us to consider the case when the strong solution explodes at a finite time. We use the latter result to construct continuous state branching processes with immigration and competition in a Lévy random environment as a strong solution of a stochastic differential equation.

Spectrally negative Lévy processes with Parisian reflection below and classical reflection above

José-Luis Pérez-Garmendia (CIMAT Guanajuato)

We consider a company that receives capital injections so as to avoid ruin. Differently from the classical bail-out settings where the underlying process is restricted to stay at or above zero, we study the case bail-out can only be made at independent Poisson times. Namely, we study a version of the reflected process that is pushed up to zero only on Poisson observation times at which the process is below zero. We also study the case with additional classical reflection above so as to model a company that pays dividends according to a barrier strategy. Focusing on the spectrally negative Lévy case, we compute, using the scale function, various fluctuation identities including capital injections and dividends.

On weak approximation of BSDEs driven by Lévy processes

Martijn Pistorius (Imperial College London)

In this talk we present a weak approximation scheme for a class of BSDEs driven by a Wiener process and an (in)finite activity Poisson random measure. The approximating backward stochastic difference equations (BS Δ Es) are driven by random walks that weakly approximate the given Wiener process and Poisson random measure. We establish the weak convergence to the solution of the BSDE and the numerical stability of the sequence of solutions of the BS Δ Es. By way of application, we present a functional limit theorem for a subclass arising in a risk modelling application. This talk is based on joint work with D. Madan and M. Stadje.

Williams decomposition for superprocesses

Yanxia Ren (Peking University)

We are interested in a spinal decomposition for superprocesses involving the ancestral lineage of the last individual alive (Williams' decomposition).

For superprocesses with homogeneous branching mechanism, the spatial motion is independent of the genealogical structure. As a consequence, the law of the ancestral lineage of the last individual alive does not distinguish from the original motion. Therefore, in this setting, the description of $X^{(h_0)}$ may be deduced from Abraham and Delmas (2009) where no spatial motion is taken into account.

For nonhomogeneous branching mechanisms on the contrary, the law of the ancestral lineage of the last individual alive should depend on the distance to the extinction time h_0 . Using the Brownian snake, Delmas and Hénard (2013) provide a description of the genealogy for superprocesses with the following non-homogeneous branching mechanism

$$\psi(x, z) = a(x)z + \beta(x)z^2$$

with the functions *a* and β satisfying some conditions.

We would like to find conditions such that the Williams' decomposition works for superprocesses with general non-homogeneous branching mechanisms. The conditions should be easy to check and satisfied by a lot of superpossess. The talk is based on a working paper with Renming Song and Rui Zhang.

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Stochastic integration with respect to cylindrical Lévy processes

Markus Riedle (King's College London)

Cylindrical Lévy processes are a natural generalisation of cylindrical Wiener processes and Gaussian white noise. However, since a cylindrical Lévy process does not enjoy a cylindrical version of the semi-martingale decomposition, one cannot apply one of the standard approaches to define stochastic integrals with respect to cylindrical Lévy processes.

In this talk, we will introduce a completely novel approach to stochastic integration. In this approach the integrator is not decomposed into a martingale and a bounded variation process. As a consequence, the sequence of stochastic integrals for simple integrands can only be considered as a sequence in the space L^0 of Hilbert space valued random variables. Convergence is established by tightness arguments utilising an approach called decoupled tangent sequences.

This talk is based on a joint work with Adam Jakubowski.

Some fluctuation theory results for Markov additive processes and its applications to self-similar Markov processes

Víctor Rivero (CIMAT Guanajuato)

In recent years there has been a renewed interest in the theory of Markov additive processes (MAPS), which is due to the large variety of topics where these processes are relevant. The theory of self-similar processes is one of them, which are connected to MAPS via the so-called Lamperti transformation, as has been shown recently by Alili, Chaumont, Grackzyk and Zak. In this talk we will explain how the theory of exit systems of Maisonneuve allows to provide a systematic approach to the fluctuation theory of both Lévy and Markov additive processes. Then we will apply some of the results obtained to the theory of self-similar Markov processes, and in particular to stable processes. This talk is based on work in progress in collaboration with Chaumont, Kyprianou, Şengül and Satitkanitkul.

Markov chain approximation of pure jump processes

Nikola Sandrić (University of Zagreb)

Let $\{X^n\}_{n \in \mathbb{N}}$ be a sequence of Markov chains on $n^{-1}\mathbb{Z}^d$, and let *X* be a Markov process on \mathbb{R}^d . The following two questions naturally arise:

- 1. When does $\{X^n\}_{n \in \mathbb{N}}$ converge weakly to a Markov process?
- 2. Can *X* be approximated (in the sense of weak convergence) by a sequence of Markov chains?

These two problems are very well investigated in the case of diffusion processes and symmetric pure jump Markov processes (generated by a symmetric nonlocal Dirichlet form). In this talk, we focus on a class of pure jump Markov processes generated by: (i) a nonsymmetric nonlocal Dirichlet form or (ii) semimartingale characteristics, and we discuss conditions for weak convergence of a sequence of Markov chains to a process in this class and Markov chain approximation of processes in this class.

Joint work with Ante Mimica and René L. Schilling

Bernstein-gamma functions and exponential functionals of Lévy processes

Mladen Savov (Bulgarian Academy of Sciences)

In this talk we consider a recurrent equation of the type

$$f(z+1) = \frac{-z}{\Psi(-z)}f(z)$$

on a subset of the imaginary line where Ψ is a negative definite function. Using the analytical Wiener-Hopf method we furnish the solution to this equation in a three term product involving the solutions to the equation $W_{\phi}(z+1) = \phi(z)W_{\phi}(z)$ on $\{z \in \mathbb{C} : \Re(z) > 0\}$, where ϕ is any Bernstein function. The functions W_{ϕ} we call the Bernstein-Gamma functions and note that they have appeared earlier in some studies albeit in more restricted context. Via a couple of computable parameters depending on ϕ we provide a thorough characterization of W_{ϕ} as a meromorphic function on an identifiable complex strip. Moreover, we establish universal Sterling type asymptotic which is explicit in terms of ϕ . This allows the thorough understanding of the decay of |f(z)| at least along the imaginary lines $z = a + i\mathbb{R}$, $a \in (0, 1)$, and an access to quantities important for many theoretical and applied studies in probability theory and analysis.

The foremost motivation for the aforementioned results is their application to the study of an important class of non-self-adjoint Markov processes. However, in this talk, as an application, we present some general results on the law of the exponential functional of Lévy processes, that is $\int_0^\infty e^{-\xi_s} ds$, which are a consequence of the understanding of f and the fact that the Mellin transform of the exponential functional satisfies the recurrent equation $f(z+1) = \frac{-z}{\Psi(-z)}f(z)$. We discuss results such as smoothness, large and small asymptotic, expansions, bounds and Mellin Barnes representations. In a comparatively general context we also study the weak convergence of the suitably scaled in space and time probability measures of $\int_0^t e^{-\xi_s} ds$ as $t \to \infty$ and $\int_0^\infty e^{-\xi_s} ds = \infty$. The derivation of all our results and applications relies crucially on a mixture of analytical, complex-analytical and probabilistic techniques which we will try to highlight during the talk.

Joint work with Pierre Patie (Cornell)

Entrance laws at the origin of self-similar Markov processes in \mathbb{R}^d

Batı Şengül

(University of Bath)

In this talk, we consider self-similar Markov processes defined on \mathbb{R}^d without the origin, which are killed upon hitting the origin. The goal is to try to take a weak limit as $x \to 0$ under mild assumptions. The process started at the origin is obtained in a unique way by conditioning the process to be continuously absorbed at the origin and then reversing time from the absorption time. The proof uses recent techniques in Markov additive process and the Lamperti-Kiu transformation. This is joint work with Loïc Chaumont, Andreas Kyprianou and Víctor Rivero.

Lévy-driven CARMA processes: Non-equidistant observations and local stationarity

Robert Stelzer (Ulm University)

Continuous time autoregressive moving averages driven by Lévy processes are a fundamental time series model defined in continuous time. Such models are in particular relevant in the context of high frequency data or irregularly sampled data, for instance. In this talk we first give an introduction to Lévydriven CARMA processes and the well-known statistical estimation procedures based on equidistantly sampled observations.

Then we turn to the question of estimating the parameters of the process based on non-equidistant observations. We present and discuss an approach for observations made at independent Poissonian times where it can be shown that identification of parameters is actually less problematic than in the case of equidistant observations. In this set-up a Whittle-type estimator for the parameters can be used which is consistent and asymptotically normal under certain conditions. As an alternative approach we consider path-wise Fourier transforms of CARMA processes and high-frequency limits along non-equidistant deterministic grids as a first step to obtaining alternative estimators. Finally, as time permits, we propose a definition of local stationarity for (Lévy-driven) processes in continuous time. Based on this we discuss, how to define CARMA processes with time-varying autoregressive and moving average parameters such that they are locally stationary.

This talk is based on joint work with Frank Bosserhoff, Żywilla Fechner and Annemarie Bitter.

Optimal importance sampling for Lévy processes

Peter Tankov (Université Paris 7)

We develop generic and efficient importance sampling estimators for Monte Carlo evaluation of prices of single- and multi-asset European and certain exotic options in asset price models driven by Lévy processes, extending ealier works which focused on the Black-Scholes and continuous stochastic volatility models [1,3]. Using recent results from the theory of large deviations on the path space for processes with independent increments [2], we compute an explicit asymptotic approximation for the variance of the pay-off under an Esscher-style change of measure. Minimizing this asymptotic variance, we then obtain an explicit asymptotically efficient importance sampling estimator of the option price. Numerical tests show variance reduction factors ranging between 4 and 25 for European options in the variance gamma model with parameters calibrated to market data.

Joint work with Adrien Genin.

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On splitting trees and Lévy processes

Gerónimo Uribe Bravo (UNAM Mexico)

Splitting trees are the class of random trees featuring a self-similarity property where individuals give birth, at constant rate during their lifetimes, to iid copies of themselves. These random trees thus have a chronological interpretation. We first provide an adequate space of locally compact trees within which trees can be represented by sequences of stochastic processes satisfying a compatibility condition. In the case of splitting trees, we then characterize these sequences in terms of reflected Lévy processes. Finally, we introduce the genealogy associated to splitting trees and obtain Ray-Knight type theorems which extend the ones for (sub)critical Lévy trees to the supercritical case.

The absorption problem for multidimensional random walks

Vladislav Vysotsky (Imperial College London)

Consider the probability that the convex hull of a random walk in \mathbb{R}^d does not absorb the origin by the time *n*, which in dimension one means that there is no sign change. The remarkable formula of Sparre Andersen (1949) states that any random walk with a symmetric density of increments stays positive with the same probability (2n-1)!!/(2n)!! regardless of the density. We prove a multidimensional distribution-free counterpart of this result and provide an explicit tractable formula for the absorption probability. This formula is then used to study the asymptotic of the absorption probability in a fixed dimension and in the high-dimensional setting. Our idea is to show that the absorption problem is equivalent to a geometric problem on counting the number of Weyl chambers in \mathbb{R}^n intersected by a generic linear subspace of codimension d. This method also applies to convex hulls of random walk bridges, and to the joint convex hulls of several symmetric random walks. In particular, we recover the Wendel formula for the absorption probability of the convex hull of i.i.d. random vectors in \mathbb{R}^d with a symmetric density. This is a joint work with Zakhar Kabluchko (Munster) and Dmitry Zaporozhets (St. Petersburg).

First-passage times for random walks with non-identically distributed increments

Vitali Wachtel (University of Augsburg)

We consider a random walk S_n with independent but not necessarily identically distributed increments. Assuming that increments satisfy the Lindeberg condition, we investigate the tail-behaviour of the time $T_x = \min\{n : x + S_n \le 0\}$. We also prove limit theorems for S_n conditioned on $T_x > n$.

Fragmentation with growth

Alex Watson (University of Manchester)

In models of fragmentation with growth, one has a number of independent cells, each of which grows continuously in time until a fragmentation event occurs, at which point the cell splits into two or more child cells of a smaller mass. Each of the children is independent and behaves in the same way as its parent. The rate of fragmentation may be infinite, and fragmentation may be homogeneous (where the rate does not depend on the mass of the cell) or self-similar (where the rate is a power of the mass). We will discuss some recent results, with a focus on solutions of the growth-fragmentation equation from a probabilistic perspective.

On the refracted-reflected spectrally one-sided Lévy processes

Kazutoshi Yamazaki (Kansai University)

We study a combination of the refracted and reflected Lévy processes. Given a spectrally one-sided Lévy process and two boundaries, it is reflected at the lower boundary while, whenever it is above the upper boundary, a linear drift at a constant rate is subtracted from the increments of the process. Using the scale functions, we compute the resolvent measure, the Laplace transform of the occupation times as well as other fluctuation identities. Several applications are also discussed. Joint work with José Luis Pérez.

Generalization of refracted Lévy processes and its application to exit problems

Kouji Yano (Kyoto University)

Kyprianou-Loeffen have introduced refracted Lévy processes and studied their exit problems. They are defined via stochastic differential equations as a Markov process which behaves as a spectrally negative Lévy process with different drifts above and below a fixed level.

In this talk we generalize Kyprianou–Loeffen's refracted Lévy processes. Our process behaves as spectrally negative Lévy processes with different drifts and different Lévy measures above and below a fixed level. We construct our process based on the excursion theory.

This talk is based on a joint work with Kei Noba (Kyoto University).

Sample-path large deviations for heavy-tailed random walks and Lévy processes

Bert Zwart

(Technische Universiteit Eindhoven)

Many rare events in man-made networks exhibit heavy-tailed features. Examples are file sizes, delays and financial losses, but also magnitudes of systemic events, such as the size of a blackout in a power grid. The theory of rare events in the heavy-tailed case is, apart from a few isolated examples, restricted to events that are caused by a single big jump. In this work we develop samplepath large deviations for random walks and Lévy processes in the heavy-tailed case that go beyond such restrictions.

Joint work with Jose Blanchet (Columbia University) and Chang-Han Rhee (CWI).

POSTERS

Estimates of Dirichlet heat kernel for symmetric Markov processes

Kyung-Youn Kim (Technische Universität Dresden)

We consider a large class of symmetric pure jump Markov processes dominated by isotropic unimodal Lévy processes with weak scaling conditions.

We first establish sharp two-sided heat kernel estimates for these processes in $C^{1,\rho}$ open sets, $\rho \in (\overline{\alpha}/2, 1]$ where $\overline{\alpha}$ is the upper scaling parameter in the weak scaling conditions. As a corollary of our main result, we obtain a sharp two-sided Green function estimates and a scale invariant boundary Harnack inequality with explicit decay rates in $C^{1,\rho}$ open sets.

This is based on the joint work with Tomasz Grzywny and Panki Kim.

The sensitivity of the solution to a Lévy driven SDE with respect to perturbations of the noise

Tania Kosenkova (Potsdam University)

We provide an estimate for the Wasserstein-Kantorovich-Rubinstein distance on path space between two Lévy diffusions in terms of the Lévy triplets. The limit theorems with such estimates on the rate of convergence give the opportunity to obtain a goodness-of-fit tests for models with Lévy noise. Considered as an example are low-dimensional conceptual climate models with paleoclimatic data.

Existence of Feller processes: Parametrix construction

Franziska Kühn (Technische Universität Dresden)

We present a new existence result for Feller processes. Feller processes behave locally like Lévy processes, but the Lévy triplet may depend on the current position. They can be characterized by their, so-called, symbol; this is the analogue of the characteristic exponent (log characteristic function) in the Lévy case.

Using a parametrix construction, we prove the existence of Feller processes with a given symbol under weak assumptions on the regularity (with respect to the space variable *x*) of the symbol. We derive heat kernel estimates for the transition density as well as its derivatives, and prove the well-posedness of the corresponding martingale problem. Our result applies to symbols of the form $q(x,\xi) = f_{\alpha(x)}(|\xi|^2)$ where $(f_{\alpha})_{\alpha \in I}$ is a family of Bernstein functions and $\alpha : \mathbb{R}^d \to I$ a Hölder continuous mapping. This includes in particular stablelike, relativistic stable-like and normal tempered stable-like processes. In dimension d = 1 we also obtain results for solutions of SDEs driven by Lévy processes.

Space-time fractional Dirichlet problems

Tomasz Luks (Paderborn University)

Fractional derivatives, invented by Leibniz in 1695, have recently found new applications in many branches of science and engineering. From practical point of view, it is often necessary to use numerical methods to find the solutions of time-fractional partial differential equations. Many effective numerical schemes have been developed to solve fractional partial differential equations on bounded domains. However, the theory justifying that these problems are well posed has been lacking. In this work we take a step in that direction by establishing explicit solutions to a broad class of time-fractional Cauchy processes $\partial_t^{\beta} u(x, t) = Lu(x, t)$, u(x, 0) = f(x) on a regular bounded domain, where ∂_t^{β} is the Caputo fractional derivative of order $0 < \beta < 1$, and *L* is the infinitesimal generator of a Markovian semigroup. The solutions are obtained by the use of the corresponding Markov processes time-changed by an inverse stable subordinator

whose index equals the order of the fractional time derivative. Applications include the infinitesimal generators of semigroups associated with Lévy processes whose probability transition functions are absolutely continuous with respect to the Lebesgue measure.

It is a joint work with Boris Baeumer from the University of Otago and Mark M. Meerschaert from Michigan State University.

Generalization of refracted Lévy processes and its application to exit problems

Kei Noba (Kyoto University)

Generalizing Kyprianou-Loeffen's refracted Lévy processes, we define a new refracted Lévy process which is a Markov process whose behaviors during the period of non-negative values and during that of negative ones are Lévy processes different from each other. To construct it, we utilize the excursion theory. We study its exit problem and the potential measures of the killed processes.

This poster is based on a joint work with Kouji Yano (Kyoto University).

Reflected Brownian motion on nested fractals

Mariusz Olszewski (Wrocław University of Technology)

Dealing with several problems of quantum physics related to the potential theory leads to analyzing Brownian motion on unbounded nested fractals and bringing the case to the process on a compact set by particular reflection.

We examine in which cases the reflected Brownian motion on nested fractals can be constructed as a strong Markov process and show steps of this construction. The construction is a generalization of the one made on Sierpiński gasket in [1].

It is a joint work with Kamil Kaleta and Katarzyna Pietruska-Pałuba.

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Multi-type continuous state branching processes

Sandra Palau Calderón (CIMAT Guanajuato)

A multi-type continuous state branching process with a countable infinity number of types is defined as a super Markov chain with both local and nonlocal branching mechanism. We study the events of local and global extinction. In particular, a local extinction dychotomy is given.

Recurrent extensions of real self-similar Markov processes

Henry Pantí (UADY)

An open question raised by Lamperti (1972) is how to extent a positive self similar Markov processes after reaching the point zero. He answered this question in the Brownian case. Years later Vuolle-Apiala (1994) established conditions for the existence of an entrance law and the existence and uniqueness of an extension for a positive self similar Markov processes. Fitzsimmons (2006), Rivero (2005, 2007) gave a simpler condition: A positive self-similar Markov process that hits 0 in a finite time admits a self-similar recurrent extension that leaves 0 continuously if and only if the Lévy process in the Lamperti transformation satisfies Cramér's condition. Recently in Chaumont et al (2013) is established the Lamperti transformation for real self similar Markov processes. Associated with real self similar Markov processes via the Lamperti transform are the so called Markov Additive Processes (MAPs for short). The latter allows us generalize Rivero's work to real valued case. Our main result ensures that a real self-similar Markov process with a finite hitting time of the point zero has a recurrent extension that leaves 0 continously if and only if the MAP associated, via Lamperti transformation, satisfies the Cramér condition. We study two examples: the stable process and the stable process conditioned to continuously absorb at the origin (Kyprianou et al (2015)).

Joint work with J.C. Pardo and V. Rivero.

Transition density estimates for Brownian motion in a ball

Grzegorz Serafin (Wrocław University of Technology)

Let $g_n(t, x, y) = (2\pi t)^{n/2} e^{-|x-y|^2/2t}$ be the standard transition density of *n*-dimensional Brownian motion, and let $g_n^D(t, x, y)$ be the the density of the process killed when exiting the set $D \subset \mathbb{R}^n$. In 2002, Qi S. Zhang has proven that for any bounded $\mathscr{C}^{1,1}$ set *D* there exists C > 1 such that

$$\begin{aligned} \frac{1}{C} \left(1 \wedge \frac{\delta(x)\delta(y)}{t} \right) (2\pi t)^{n/2} e^{-c_1|x-y|^2/2t} &\leq g_n^D(t, x, y) \\ &\leq C \left(1 \wedge \frac{\delta(x)\delta(y)}{t} \right) (2\pi t)^{n/2} e^{-c_2|x-y|^2/2t} \end{aligned}$$

where $x, y \in D$, t < T and $\delta(x)$ denotes the distance between the point x and the boundary of D. In fact, the upper bound is due to E. B. Davies (1987). It appears that even for convex sets the constants c_1 and c_2 above do not have to be the same. However, we may achieve this, at least in some cases, changing other factors, e.g. we have [1]

$$\begin{split} \frac{1}{C} \Big(1 \wedge \frac{xy}{t} \Big) \Big(1 \wedge \frac{(1-x)(1-y)}{t} \Big) g_1(t,x,y) &\leq g_1^{(0,1)}(t,x,y) \\ &\leq C \Big(1 \wedge \frac{xy}{t} \Big) \Big(1 \wedge \frac{(1-x)(1-y)}{t} \Big) g_1(t,x,y) \end{split}$$

where $x, y \in (0, 1)$, t < T. Our goal is to provide analogous two-sided estimates for the transition density of the *n*-dimensional ball such that constants in exponents in both lower and upper bound are the same. Note that large-time estimates are well known.

References

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Generalised Langevin equation with Lévy distribution

Jakub Ślęzak (Wrocław University of Technology)

Stochastic processes which exhibit non-Gaussian infinitely distributions attract continuous attention in empirical sciences and modelling. However, the possible physical origin of such distributions is largely shrouded in mystery. Here, we study one of the most strictly derived statistical models, the Generalised Langevin equation with hope that it can further link the class of Lévy processes and distributions with statistical physics. In order to archive that we look into mechanical origins of the motion in complex environments and study the assumptions that lead to the Lévy distributions, as well as provide a description of the class of resulting processes.

Approximation of eigenfunction of fractional Laplacian

Grzegorz Żurek (Wrocław University of Technology)

During this presentation it will be shown that method presented by Mateusz Kwaśnicki (2012) is good to obtain estimates of eigenfunctions of the fractional Laplace operator. We will show a modification of this method which allows us to obtain estimates of eigenfunctions for different sets.

We will also present results obtained for either one and two dimensional simple domains, the pace of convergence to the boundary limits and some results which are treating about concavity.

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